Spiral plat sans courbes terminales

Anisochronisme causé par l'affaissement du spiral en position verticale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-6}$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L := L_{sp} \quad L = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56\times 10^3\,deg$

Positions du piton $\quad r_P := 0.5\cdot d2_{sp} \quad \boxed{\alpha_P := 90\cdot deg} \quad x_P := r_P\cdot cos(\alpha_P) \quad y_P := r_P\cdot sin(\alpha_P)$

$x_P = 0\,mm \quad y_P = 2.26\,mm \quad z_P := x_P + i\cdot y_P$

Position du point d'attache à la virole $\quad r_V := 0.5\cdot d1_{sp} \quad \alpha_V(\theta) := \psi_0 + \theta + \alpha_P$

$x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta)) \quad z_V(\theta) := x_V(\theta) + i\cdot y_V(\theta)$

Forme initiale du spiral

$a := \frac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_P - a\cdot(\alpha - \alpha_P) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$z_{0s}(\alpha) := r_s(\alpha)\cdot exp(i\cdot\alpha)$

$s(\alpha) := r_P\cdot(\alpha - \alpha_P) - \frac{a}{2}\cdot(\alpha - \alpha_P)^2 \quad s(\psi_0 + \alpha_P) = 11.182\,cm$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Déformation du spiral sous son propre poids

Calculs des réactions et du moment fléchissant

Moment fléchissant du spiral avec le point d'attache libre

$s\xi(\alpha) := \int_{\alpha_P}^{\alpha} x_{0s}(\alpha')\cdot r_s(\alpha')\,d\alpha' \quad \xi_{0s} := \frac{1}{L}\cdot s\xi(\psi_0) \quad \eta_{0s} := \frac{1}{L}\cdot\int_{\alpha_P}^{\psi_0} y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha$

$s\xi_1(\alpha) := (2\cdot a^2\cdot\alpha - 2\cdot a^2\cdot\alpha_P - 2\cdot r_P\cdot a)\cdot cos(\alpha)$

$s\xi_2(\alpha) := (r_P^2 - 2\cdot a^2 + a^2\cdot\alpha^2 + a^2\cdot\alpha_P^2 - 2\cdot r_P\cdot a\cdot\alpha + 2\cdot r_P\cdot a\cdot\alpha_P - 2\cdot a^2\cdot\alpha_P\cdot\alpha)\cdot sin(\alpha)$

$s\xi_3(\alpha) := 2\cdot a^2\cdot sin(\alpha_P) + 2\cdot r_P\cdot a\cdot cos(\alpha_P) - r_P^2\cdot sin(\alpha_P) \quad s\xi(\alpha) := s\xi_1(\alpha) + s\xi_2(\alpha) + s\xi_3(\alpha)$

$M_{fg}(\alpha) := \frac{m_s\cdot g}{L}\cdot\left[(L - s(\alpha))\cdot x_{0s}(\alpha) - L\cdot\xi_{0s} + s\xi(\alpha)\right]$

Calcul des réactions au point d'attache

$$q2_{0s} := \frac{1}{L}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} y_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad p2_{0s} := \frac{1}{L}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad q2_{0s} = 1.226\,mm^2$$

$$p2_{0s} = 1.479\,mm^2$$

$$k_{0s} := \frac{1}{L}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha \qquad k_{0s} = -0.2\,mm^2$$

$$v_g := \frac{-1}{E\cdot I_{33}}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} x_{0s}(\alpha)\cdot M_{fg}(\alpha)\cdot r_s(\alpha)\,d\alpha \qquad u_g := \frac{1}{E\cdot I_{33}}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} y_{0s}(\alpha)\cdot M_{fg}(\alpha)\cdot r_s(\alpha)\,d\alpha$$

$$v_g = -0.061\,mm \qquad u_g = -5.655\times 10^{-3}\,mm$$

$$\mathbf{S_g} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} -k_{0s} & p2_{0s} \\ q2_{0s} & -k_{0s}\end{pmatrix} \qquad \mathbf{R_g} := -\mathbf{S_g}^{-1}\cdot\begin{pmatrix} v_g \\ u_g\end{pmatrix} \qquad \mathbf{R_g} = \begin{pmatrix} -1.342\times 10^{-6} \\ 2.614\times 10^{-5}\end{pmatrix} N \qquad \left|\mathbf{R_g}\right| = 2.617\times 10^{-5}\,N$$

Approximations

$$P2_{0s} := \frac{1}{L^2}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} s(\alpha)\cdot x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad K_{0s} := \frac{1}{L^2}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} s(\alpha)\cdot x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha$$

$$P2_{0s} = 0.484\,mm^2 \qquad K_{0s} = -0.088\,mm^2$$

$$v_{ga} := \frac{L\cdot m_s\cdot g}{E\cdot I_{33}}\cdot\left[\left(-p2_{0s} + P2_{0s}\right) + \frac{1}{2}\cdot\xi_{0s}^{\ 2}\right] \qquad v_{ga} = -0.061\,mm$$

$$u_{ga} := \frac{L\cdot m_s\cdot g}{E\cdot I_{33}}\cdot\left(k_{0s} - K_{0s} - \xi_{0s}\cdot\eta_{0s} + \frac{1}{L^2}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} y_{0s}(\alpha)\cdot s\xi(\alpha)\cdot r_s(\alpha)\,d\alpha\right) \qquad u_{ga} = -5.643\times 10^{-3}\,mm$$

$$\sigma2 := \frac{1}{2}\cdot\left(r_P^{\ 2} + r_V^{\ 2}\right) \qquad \sigma2 = 2.705\,mm^2 \qquad p2_{0a} := \frac{1}{2}\cdot\sigma2 \qquad p2_{0a} = 1.353\,mm^2$$

$$P2_{0a} := \frac{r_P^{\ 2} + 2\cdot r_V^{\ 2}}{12} \qquad R_{gx} := -\frac{4}{5}\cdot m_s\cdot g\cdot\frac{r_P^{\ 5}}{L\cdot\left(r_P^{\ 4} - r_V^{\ 4}\right)} \qquad R_{gy} := \frac{m_s\cdot g}{3}\cdot\frac{2\cdot r_P^{\ 2} + r_V^{\ 2}}{r_P^{\ 2} + r_V^{\ 2}}$$

$$P2_{0a} = 0.476\,mm^2 \qquad R_{gx} = -6.285\times 10^{-7}\,N \qquad R_{gy} = 2.51\times 10^{-5}\,N$$

Moment fléchissant du spiral avec le point d'attache lié à la virole

$$M_f(\alpha) := r_s(\alpha)\cdot\left[\sin(\alpha)\cdot\mathbf{R_g}_0 + \left[\frac{m_s\cdot g}{L}\cdot(L - s(\alpha)) - \mathbf{R_g}_1\right]\cdot\cos(\alpha)\right] - \frac{m_s\cdot g}{L}\cdot\left(L\cdot\xi_{0s} - s\xi(\alpha)\right)$$

Calcul de la déformation du spiral

Spiral non lié à la virole

$$\Delta\varphi_g(\alpha) := \frac{1}{E\cdot I_{33}}\cdot\int_{\alpha_P}^{\alpha} M_{fg}(\alpha')\cdot r_s(\alpha')\,d\alpha' \qquad \Delta\varphi_{IV} := \Delta\varphi_g\left(\psi_0 + \alpha_P\right) \qquad \Delta\varphi_{IV} = -0.151\,deg$$

$$z_g(\alpha) := z_{0s}(\alpha) - \frac{i}{E\cdot I_{33}}\cdot\int_{\alpha_P}^{\alpha}\left(z_{0s}(\alpha') - z_{0s}(\alpha)\right)\cdot M_{fg}(\alpha')\cdot r_s(\alpha')\,d\alpha'$$

Spiral lié à la virole

$$\Delta\varphi_{12}(\beta) := \frac{1}{E \cdot I_{33}} \cdot \int_{\alpha_P}^{\beta} \left[sin(\alpha) \cdot \boldsymbol{R}_{\boldsymbol{g}_0} + \left[\frac{m_s \cdot g}{L} \cdot (L - s(\alpha)) - \boldsymbol{R}_{\boldsymbol{g}_1} \right] \cdot cos(\alpha) \right] \cdot r_s(\alpha)^2 \, d\alpha$$

$$\Delta\varphi_{11}(\beta) := \frac{1}{E \cdot I_{33}} \cdot \int_{\alpha_P}^{\beta} \frac{m_s \cdot g}{L} \cdot \left(L \cdot \xi_{0s} - s\xi(\alpha) \right) \cdot r_s(\alpha) \, d\alpha \qquad \Delta\varphi_1(\beta) := \Delta\varphi_{12}(\beta) - \Delta\varphi_{11}(\beta)$$

$$z_1(\alpha) := z_{0s}(\alpha) - \frac{i}{E \cdot I_{33}} \cdot \int_{\alpha_P}^{\alpha} \left(z_{0s}(\alpha') - z_{0s}(\alpha) \right) \cdot M_f(\alpha') \cdot r_s(\alpha') \, d\alpha'$$

Graphes de la déformation

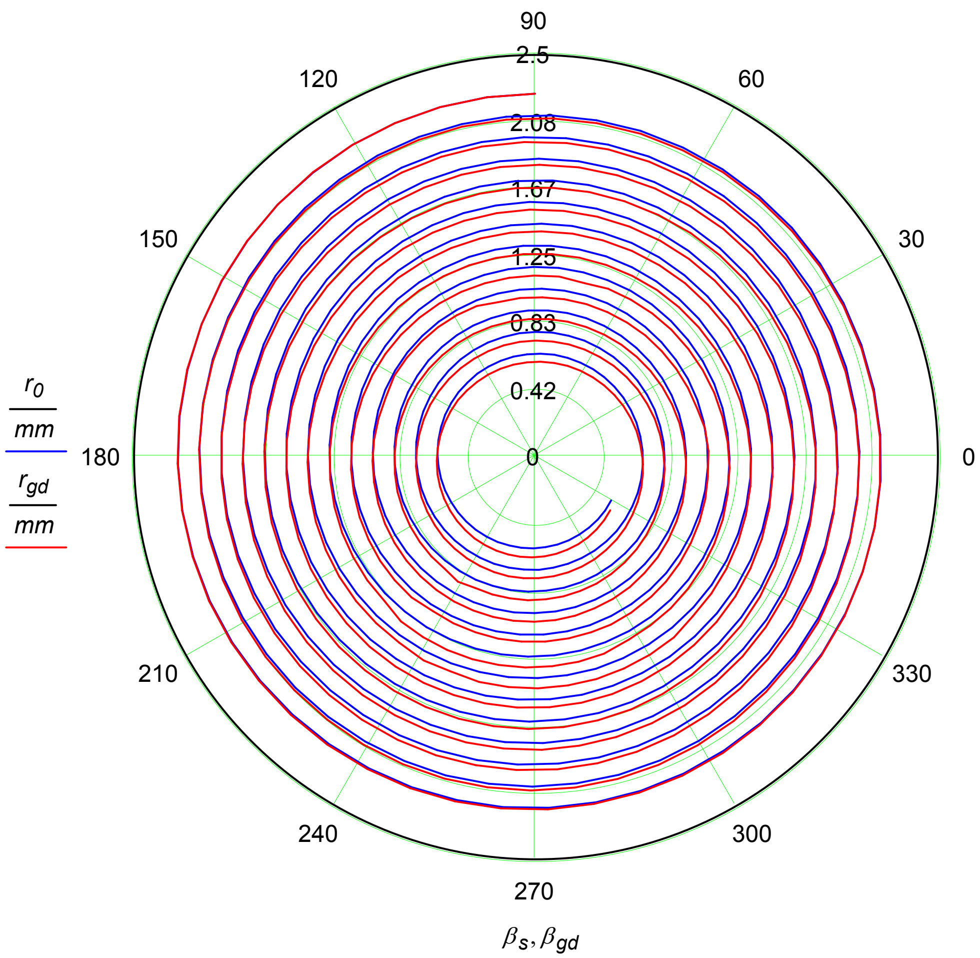
Forme naturelle $\quad n := 50 \cdot partentière\left(n_{sp}\right) + 1 \quad i := 0 .. n-1 \quad \Delta\alpha := \dfrac{\psi_0}{n-1} \quad \alpha_i := i \cdot \Delta\alpha + \alpha_P$

$$x_{0_i} := x_{0s}\left(\alpha_i\right) \quad y_{0_i} := y_{0s}\left(\alpha_i\right) \quad r_0 := \overrightarrow{\sqrt{{x_0}^2 + {y_0}^2}} \quad \beta_s := \overrightarrow{Atan\left(x_0, y_0\right)}$$

Déformée gravitationnelle libre

$$z_{gd_i} := z_g\left(\alpha_i\right) \quad n_{pt} := dernier\left(z_{gd}\right) \quad x_{gd} := Re\left(z_{gd}\right) \quad y_{gd} := Im\left(z_{gd}\right) \quad r_{gd} := \overrightarrow{\left|z_{gd}\right|} \quad r_{gd_{npt}} = 0.578\,mm$$

$$\beta_{gd} := \overrightarrow{Atan\left(x_{gd}, y_{gd}\right)} \quad \beta_{gd_0} = 90\,deg \quad \beta_{gd_{npt}} = 324.35\,deg \quad mod\left(\alpha_V(0), 2 \cdot \pi\right) = 330\,deg$$

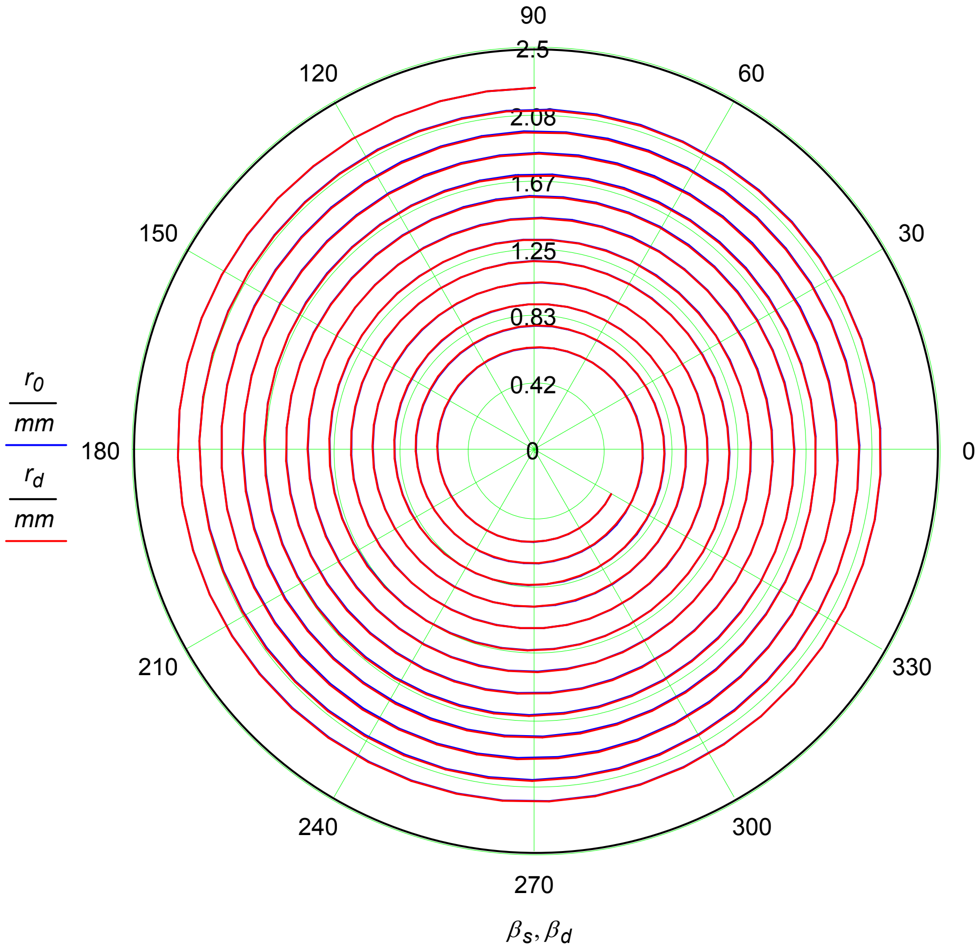


$x_V(0) = 0.476\,mm$ $\quad x_{gd_{npt}} - x_V(0) = -6.381 \times 10^{-3}\,mm$ $\quad y_V(0) = -0.275\,mm$ $\quad y_{gd_{npt}} - y_V(0) = -0.062\,mm$

Déformée avec liaison au point d'attache

$z_{d_i} := z_1(\alpha_i)$ $\quad n_{pt} := dernier(z_d)$ $\quad x_d := Re(z_d)$ $\quad y_d := Im(z_d)$ $\quad r_d := \overrightarrow{|z_d|}$ $\quad r_{d_{npt}} = 0.55\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 90\,deg$ $\quad \beta_{d_{npt}} = 329.961\,deg$ $\quad mod(\alpha_V(0), 2 \cdot \pi) = 330\,deg$



$x_{d_{npt}} - x_V(0) = -1.886 \times 10^{-4}\,mm$ $\quad y_{d_{npt}} - y_V(0) = -3.267 \times 10^{-4}\,mm$

Perturbation de marche causée par l'affaissement du spiral

Perturbation de marche en fonction de son élongation en position horiszontale

$$\mathbf{\Delta 1}(\theta) := \frac{i \cdot \theta}{L} \cdot \int_{\alpha_P}^{\psi_0 + \alpha_P} z_{0s}(\alpha) \cdot exp\left(i \cdot \theta \cdot \frac{s(\alpha)}{L}\right) \cdot r_s(\alpha)\, d\alpha \qquad \mathbf{\Delta 1}(\theta_0) = -0.01 - 0.207i\,mm$$

$$X_H(\theta) := \frac{\left(\left|\mathbf{\Delta 1}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_H(\theta) := \frac{d}{d\theta} X_H(\theta) \qquad \delta_H(\theta_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2 \cdot \pi} \gamma_H(\theta_0 \cdot cos(\varphi)) \cdot cos(\varphi)\, d\varphi$$

$$\mu_H(\theta_0) := -86400 \cdot \delta_H(\theta_0) \qquad \boxed{\mu_H(\theta_0) = 73.086}$$

Perturbation de marche en fonction de son élongation avec affaissement gravitationnel

$$\Delta\mathbf{1g}(\theta) := \frac{i\cdot\theta}{L}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} z_1(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha)}{L}\right)\cdot r_s(\alpha)\, d\alpha$$

$$\Delta\mathbf{1g}(\theta) := \frac{i}{L}\cdot\int_{\alpha_P}^{\psi_0+\alpha_P} z_1(\alpha)\cdot\theta\cdot exp\left[i\cdot\theta\cdot\frac{r_P\cdot(\alpha-\alpha_P)-\frac{a}{2}\cdot(\alpha-\alpha_P)^2}{L}\right]\cdot r_s(\alpha)\, d\alpha$$

$$X_g(\theta) := \frac{1}{\sigma 2}\cdot\left(\left|\Delta\mathbf{1g}(\theta)\right|\right)^2 \qquad \gamma_g(\theta) := \frac{d}{d\theta}X_g(\theta)$$

$$\delta_g(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_g(\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\, d\varphi$$

$k := 0..5 \qquad \theta_k := 100\cdot deg + k\cdot 50\cdot deg \qquad \mu_k := \mu_H(\theta_k) \qquad \mu_{g_k} := -86400\cdot\delta_g(\theta_k)$

$ECRIREPRN$("E:\Résonateur (TA)\datamathcad\pert_hor.prn") $:= \mu$ $\qquad n_{sp} = 12.667$

$ECRIREPRN$("E:\Résonateur (TA)\datamathcad\pert_aff.prn") $:= \mu_g$

$\mu_{hor} := LIREPRN$("E:\Résonateur (TA)\datamathcad\pert_hor.prn")

$\mu_{aff} := LIREPRN$("E:\Résonateur (TA)\datamathcad\pert_aff.prn")

$\mu_{aff}^T = (66.77 \quad 61.55 \quad 62.29 \quad 69.17 \quad 77.82 \quad 82.35) \qquad \Delta\mu_{aff} := \overrightarrow{(\mu_{aff}-\mu_{hor})}$